First test of PoQU 2018 - Early quantum theory

Please write your name and student number on this sheet (and any additional sheets).

1. In the Rutherford atomic model, the electron spins around the nucleus at a given radius. Its acceleration $a = v^2/r$ is provided by the electrostatic force $F = e^2/(4\pi\epsilon_0 r^2)$. The Bohr model imposes an additional quantization of angular momentum $L = mvr = nh/(2\pi)$.
   What are the allowed radii of the electron in the Bohr model? What is the numerical value of the smallest admissable radius?

2. Given that the binding energy of the electron in the ground state of the Hydrogen atom is -13.6 eV, what is the largest possible energy of a photon that is emitted by a quantum transition to the first excited state of the electron? What is its wavelength (approximately)?

3. Consider the Helium ion (i.e. with a positive charge of two protons) instead.
   What are the admissable energy states for an electron in this electrostatic field according to the Bohr model? How does the emission spectrum of the Helium ion compare to that of the Hydrogen atom; is one a subset of the other, or are they completely different?

If needed, use the following approximate values: $c = 3 \cdot 10^8$ m/s, $e = 2 \cdot 10^{-19}$ C, $\epsilon_0 = 9 \cdot 10^{-12}$ C^2 / (N m^2), $h = 7 \cdot 10^{-34}$ J s, $m_e = 9 \cdot 10^{-31}$ kg, 1 eV $= 2 \cdot 10^{-19}$ J.

PoQU - Test 2

Consider a particle inside the following potential (where U_0 is a constant energy term):

$$U(x) = \begin{cases} U_0, & 0 \leq x \leq l \\ \infty, & \text{otherwise} \end{cases}.$$

(1) Find ψ_1 (the ground state) and the corresponding energy. *Hint*: $H\psi_1 = E_1\psi_1$ where the *Hamiltonian* H is given by $H = K + U(x)$.

Now, we bring the infinite walls down, meaning that we change the potential into

$$U(x) = \begin{cases} U_0, & 0 \leq x \leq l \\ U_1, & \text{otherwise} \end{cases}$$

for some constant energy term $U_1 > U_0$. This will decrease the energy of the ground state of the particle.

(2) Plot the ground state of this potential.

(3) Is Δx the same in the new potential, or is it smaller or larger than in the previous case? Idem for Δp. Briefly explain your answer in both cases.

Quantum Universe test 3

1. The wavefunction for the ground state of the Hydrogen atom is

$$\Psi_{100} = \frac{1}{\sqrt{4\pi r_0^3}} e^{-r/r_0}, \tag{1}$$

where $r_0 = \frac{\epsilon_0 h^2}{\pi m_e e^2}$ is the Bohr radius. Skecth the probability distribution associated with this wavefunction and calculate the most probable value for r.

2. For the Helium atom what is the energy required to remove one electron based on the Bohr model? Explain your answer and give the energy in terms of the ionization energy of Hydrogen. The final atom is a kation and behaves like Hydrogen.

3. For a Hydrogen-like ion do the energy levels of an electron depend on all quantum numbers n, l, m_l ? If $m_l = 1$ and the ion is in its lowest possible energy configuration, what are the other quantum numbers? Specify the quantum numbers for all possible transitions of the electron to lower energy configurations.

PoQU Test 4

1. Which of the following reactions and decays are possible? For those forbidden, explain what laws are violated. For those allowed, explain which interaction is responsible.
(a) $\mu^- \rightarrow e^- + \overline{\nu}_e$ (1 point)
(b) $e^- + e^+ \rightarrow \mu^+ + \mu^-$ (1 point)
(c) $p \rightarrow e^+ + \gamma$ (1 point)

2. This is the reaction of β decay of a neutron in to a proton: $n \rightarrow p + e^- + \overline{\nu}_e$
(a) Draw the corresponding Feynmann diagram using quarks (1 point)
(b) Which interaction is responsible for this reaction? (1 point)
(b) Calculate the order of magnitude of the range of this reaction, using the mass of the mediating particle (2 points)

3. In the rare decay $\pi^+ \rightarrow e^+ + \nu_e$, what is the kinetic energy of the positron? Assume the π^+ decays from rest. Use conservation of energy and momentum to calculate the kinetic energy in terms of the rest masses. (2 points)

$h = 6.63 \times 10^{-34} Js, c = 3.00 \times 10^8 m/s, m_{\pi^+} = 139.6 MeV/c^2$,
$m_e = 0.511 MeV/c^2, m_{W^-} = 80.40 \times 10^3 MeV/c^2, 1eV = 1.6022 \times 10^{-19} J$,
neutrino's are approximately massless.

Last test of PoQU 2018 - Cosmology and astronomy

Please write your name and student number on this sheet (and any additional sheets).

1. Describe in one or two sentences which key physical effect has taken place in the Universe around $t = 1$ s after the Big Bang, around $t = 300,000$ years after the Big Bang and which is presently taking place. Indicate what the associated energy and temperature scales are (only order of magnitude required), and which fundamental force is relevant. **(3 points)**

2. While an absorption line of Calcium is normally found at a 393 nm wavelength, it is seen at 423 nm in the spectrum of another galaxy.

   (a) Do you conclude that the galaxy is moving away or towards us? **(1 point)**

   (b) Calculate the distance of the galaxy to us. Use the (non-relativistic) Doppler shift and take $H = 22km/s$ per 10^6 lightyears for the Hubble parameter. **(2 point)**

3. Consider an elevator in the gravitational field of the Earth.

   (a) Which acceleration should one give to the elevator such that the people in it experience weighlessness? **(2 points)**

   (b) Is there any experiment that can tell the difference between the previous situation (inside the gravitational field of the Earth, and which this specific accelerator) and the situation where the elevator is in outer space and is not being accelerated. Briefly explain your answer. You can take the elevator to be arbitrarily large. **(1 point)**